Large Dimensional Data in Econometrics: The Curses and Blessings

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  - Test statistic is not consistent, i.e., the power will not go to ∞ under the alternative

Examples of Large Dimensional Data

Example

Climate studies: T might be the number of time points and n the number of observation stations. $\frac{n}{T}$ is moderate.

Example

Financial data: large covariance and correlation matrices, with $n \approx 400$ financial indicators are publicly posted daily and used for value-at-risk calculations.

Example

Information Retrieval/search engines: A common search engine strategy forms huge term by document incidence matrices (n and T at least in the thousands).

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- New research areas, theory and applications, in economics, finance, and econometrics!

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- Still OK, when $\frac{n}{T} \longrightarrow 0$.
- However, when n is large and comparable with T, $\frac{n}{T} \longrightarrow c$, it is known from RMT that the sample covariance matrix is no longer a good approximation to the population covariance matrix.

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$$y_t = \beta x_t + u_t$$

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  - $\widehat{\beta} \xrightarrow{p} 0$, Identification can be achieved in panel model.

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    where $y_{it} \sim I(1)$, $x_{it} \sim I(1)$, and $u_{it} \sim I(1)$.
  - $\widehat{\beta} \xrightarrow{p} 0$, Identification can be achieved in panel model.
  - Spuriousness in time series can be removed in panel model with large n and large T.

Panel Spurious Regression

- $\widehat{\beta} = \frac{\sum_{i=1}^{n} \sum_{t=1}^{T} (x_t - \overline{x}) y_t}{\sum_{i=1}^{n} \sum_{t=1}^{T} (x_t - \overline{x})^2}.$

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- $\int_0^1 E \left[W_x^2 \left(r \right) \right] dr = \int_0^1 r dr = \frac{1}{2}, \int_0^1 E \left[\underline{W}_x^2 \left(r \right) \right] dr = \frac{1}{6}$

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- $\widehat{\beta} \xrightarrow{p} 0$ as $T \to \infty$ and follows $n \to \infty$ sequentially.
- $\widehat{\beta}$ is consistent and the panel spurious model is identified.

n-consistent

- $\sqrt{n}\widehat{\beta} = \frac{\frac{1}{\sqrt{n}}\sum_{i=1}^{n}\frac{1}{T^2}\sum_{t=1}^{T}(x_t - \overline{x})y_t}{\frac{1}{n}\sum_{i=1}^{n}\frac{1}{T^2}\sum_{t=1}^{T}(x_t - \overline{x})^2}.$

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A Less Developed Theory

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- Expand arsenal of econometric tools for thinking about these large panel models.

Factor-Loading Model

$$R_n = \Lambda F^{'} + \varepsilon$$

where R_n is a $n \times T$ matrix of asset returns, F is an $T \times k$ matrix of factors, Λ is a $n \times k$ matrix of loadings and ε is a $n \times T$ matrix of idiosyncratic errors.

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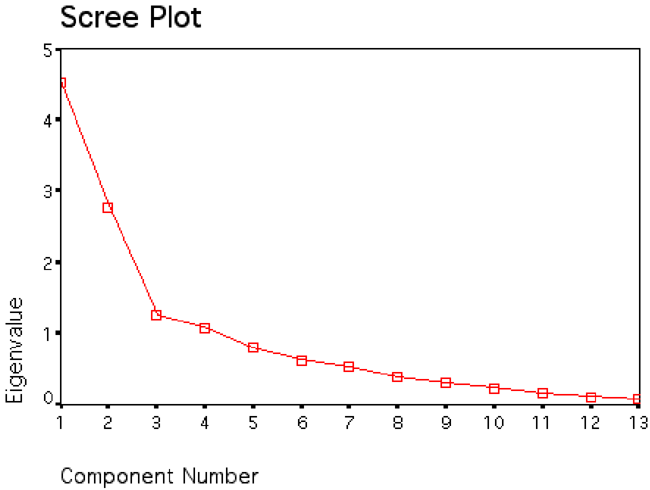
- The sample eigenvalues are more spread out than the population eigenvalues.

Scree plot



Scree Plot
5
4
3
2
1
0
Eigenvalue
1
2
3
4
5
6
7
8
9
10
11
12
13
Component Number

Dynamic Conditional Correlation (DCC) model

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- What happens if $\frac{n}{T} \longrightarrow c$?
- Test for constant conditional correlation (CCC) model.

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- Consistent Estimation with Weak Instruments in Panel Data

Change Point estimation in Time Series Set-up

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- In time series need to assume: shrinking Break: $\delta_T \to 0$ and $T^{1/2-\alpha}\delta_T \to \infty$ for some $\alpha \in \left(0, \frac{1}{2}\right)$

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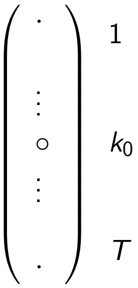
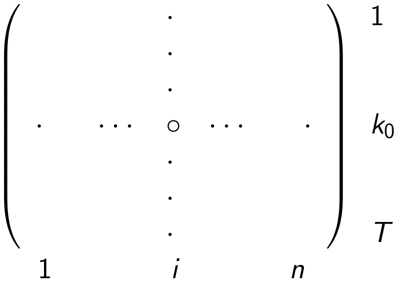


$a)$ Time series data



$b)$ Panel data

Test for Cross-Sectional Dependence in Panel Models

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- Johnstone, I. M. (2001), "On the Distribution of the Largest Eigenvalue in Principal Component Analysis," *Annals of Statistics*, 29, 295-327.

Largest Eigenvalue of Random Covariance Matrices

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- Principal component analysis (PCA). We are interested in recovering as much of the total variance in the data as possible while reducing the dimensionality of the problem from n to k.

Largest Eigenvalue of Random Covariance Matrices

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- Let $X = \left(x_1^{'}, x_2^{'}, ..., x_T^{'}\right)^{'}$ be $T \times n$ data matrix and $\frac{1}{n}X^{'}X$ be the sample covariance matrix.
- Principal component analysis (PCA). We are interested in recovering as much of the total variance in the data as possible while reducing the dimensionality of the problem from n to k.
- We are looking from k vectors $(e_1, e_2, ..., e_k)$ in R^n such that $\sum_{m=1}^{k} var\left(\langle x_i, e_m \rangle\right)$ is maximal.

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- One should choose for e the eigenvectors associated with the first k eigenvalues $\lambda_1 \geq \lambda_2 \geq ... \geq \lambda_k$ of Σ_n.

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- l_1 is an inconsistent estimator in the large n and T setting.