Testing for Cross-sectional Dependence in Fixed Effects Panel Data Models

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Cross-sectional Dependence

- Correlation between cross-sectional units:

$$y_i = \alpha + x_i' \beta + u_i; \;\; i = 1, \cdots, n.$$

  - Cross-sectional dependence (spatial depedence) in the errors: $Cov(u_i, u_j) \neq 0$

- Serial correlation in time-series:

$$y_t = \alpha + x_t' \beta + u_t; \;\; t = 1, \cdots, T.$$

  - Serial correlation: $Cov(u_t, u_s) \neq 0$

Cross-sectional Dependence

- Why?
  - Local: spatial correlation; economic distance
  - Global: common shocks
- Problems if the cross-sectional dependence was ignored
  - Efficiency loss for least squares
  - invalidates conventional t-test and F-tests

Estimation with cross-sectional Dependence

- Spatial models: Anselin (1988); Lee (2002)
- Factor Models: Bai ad Ng (2002); Bai (2003); Bai, Kao and Ng (2009); Pesaran (2006)

Testing for Cross-Sectional Dependence in Panel Models

- A fixed effects (FE) panel data model

$$y_{it} = \alpha + x_{it}'\beta + \mu_i + v_{it}, \;\; i = 1, \cdots, n; \;\; t = 1, \cdots, T. \qquad (1)$$

  - n: the dimension of cross-sectional units
  - T: the dimension of time-series
  - testing for cross-sectional dependence in the errors v_{it} with **large** n and T
- Assumption 1: Under the null, $v_{it} \sim iidN\left(0, \sigma_v^2\right)$ for all i and t.
  - the typical assumption in FE model
  - homokedasticity can be relaxed (in progress)
- $n \times 1$ vector $v_t = \left(v_{1t}, \cdots, v_{nt}\right)' \sim iidN\left(0, \Sigma_n\right)$ for all t
  - Under the null:

$$H_0 : \Sigma_n = \sigma_v^2 I_n \qquad (2)$$

  - the alternative $H_a : \Sigma_n \neq \sigma_v^2 I_n$
  - failure to reject the null: no cross-sectional dependence, nor heteroskedasticity

LM Test for Cross-Sectional Dependence

- Fixed n: Breusch and Pagan (1980) Lagrange multiplier (LM) test: under the null (assuming normality)

$$LM_{BP} = \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} T\hat{\rho}_{ij}^2 \xrightarrow{d} \chi^2_{n(n-1)/2}$$

- $\hat{\rho}_{ij}$: the sample correlation of the residuals $\widehat{v}_{it}$:

$$\hat{\rho}_{ij} = \frac{\sum_{t=1}^{T} \widehat{v}_{it}\widehat{v}_{jt}}{\left(\sum_{t=1}^{T} \widehat{v}_{it}^2\right)^{1/2} \left(\sum_{t=1}^{T} \widehat{v}_{jt}^2\right)^{1/2}}$$

- Recall for fixed n

$$T\hat{\rho}_{ij}^2 \xrightarrow{d} N(0, 1)$$

as $T \to \infty$

- Breusch, T. S. and Pagan, A. R. (1980), "The Lagrange Multiplier Test and Its Applications to Model Specification in Econometrics," *Review of Economic Studies*, 47, 239-253.

Scaled LM Test for Cross-Sectional Dependence

- Large n: LM not applicable
  - Scaled LM test

$$LM = \sqrt{\frac{1}{n(n-1)}} \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} \left(T\hat{\rho}_{ij}^2 - 1 \right)$$

  - $$\frac{LM_{BP} - E(LM_{BP})}{\sqrt{Var(LM_{BP})}} \xrightarrow{d} N(0,1)$$

    as $(T, n) \longrightarrow \infty$ sequentially
  - Since

$$E\left(T\hat{\rho}_{ij}^2\right) \simeq 1$$

    and

$$Var(LM_{BP}) \simeq n(n-1)$$

    for large T
  - However $E(T\hat{\rho}_{ij}^2 - 1)$ may not correctly centered at zero when T is small
  - scaled LM test may exhibit substantial size distortion when T is small and n is large

Pesaran CD Test

- Pesaran CD Test
  - $$\text{Pesaran s } CD = \sqrt{\frac{2T}{n(n-1)}} \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} \hat{\rho}_{ij} \xrightarrow{d} N(0,1)$$
  - $E\left(\hat{\rho}_{ij}\right) = 0$ for all n and T
  - $E\left(CD\right) = 0$
  - $\sqrt{T}\hat{\rho}_{ij} \xrightarrow{d} N(0,1)$
  - good size

Pesaran CD Test

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  - $$\text{Pesaran s } CD = \sqrt{\frac{2T}{n(n-1)}} \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} \hat{\rho}_{ij} \xrightarrow{d} N(0,1)$$
  - $E\left(\hat{\rho}_{ij}\right) = 0$ for all n and T
  - $E(CD) = 0$
  - $\sqrt{T}\hat{\rho}_{ij} \xrightarrow{d} N(0,1)$
  - good size
  - Poor power

Literature

- Econometrics: testing cross-sectional dependence in panel data
  - Pesaran (2004) *CD* test

$$\text{Pesaran } CD = \sqrt{\frac{2T}{n(n-1)}} \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} \hat{\rho}_{ij}$$

  - Pesaran, et al. (2008): bias-adjusted LM test
  - Baltagi, et al. (2003): spatial model
  - Ng (2006): spacing method
- Statistics literature: sphericity test
  - John (1972): fixed n
  - Ledoit, Wolf (2002): raw data with
  - Kapetanios (2004): raw data with heterokedasticity

Contribution

- A new test for cross-sectional dependence in the FE panel models
- Successfully control the size distortion for large n
- good power
- Applicable in micro panels

A New Test: John Test

- John Test in fixed effects panel models (1):

$$J = \frac{T}{2} tr\left[(\frac{1}{n} tr \hat{S})^{-1} \hat{S} - I_n \right]^2 - \frac{n}{2(T-1)} \tag{3}$$

$\hat{S} = \frac{1}{T}\sum_{t=1}^{T} \hat{v}_t \hat{v}_t'$: constructed by the within residuals $\hat{v}_t$

- Limiting distribution under the null:

$$J \xrightarrow{d} N(0,1)$$

as $(T, n) \to \infty$

- Desirable finite sample properties
- $\frac{n}{2(T-1)}$: due to fixed effect

Assumptions

- Assumption 1: Under the null, $v_{it} \sim iidN\left(0, \sigma_v^2\right)$ for all $i = 1, \cdots, n$ and $t = 1, \cdots, T$.
- Assumption 2: The regressors $\{x_{it}\}$ and $\{v_{it}\}$ are independent, and $E\left\|x_{it}\right\|^4 < \infty$.
- Assumption 3: $\frac{n}{T} \to c \in (0, \infty)$ as $(n, T) \to \infty$.

Limiting Distribution of the John Test under the Null

Theorem

Under Assumptions 1, 2 and 3, in the fixed effects regression model,

$$J \xrightarrow{d} N(0,1).$$

Sphericity Tests in Raw Data

$(\beta = 0)$

- Observable $v_t = (v_{1t}, \cdots, v_{nt}) \sim iidN(0, \Sigma_n)$ for all $t = 1, \cdots, T$.
- Testing for sphericity :

$$H_0 : \Sigma_n = \sigma_v^2 I_n \quad \text{or} \quad (\sigma_v^2)^{-1}\Sigma_n - I_n = 0$$

- John (1971): a test based on the statistic $\frac{nT}{2}U$, where

$$U = \frac{1}{n} tr\left((\frac{1}{n}trS)^{-1}S - I_n \right)^2 = \frac{1}{n}\left\| (\frac{1}{n}trS)^{-1}S - I_n \right\|_F^2$$

$S = \frac{1}{T}\sum_{t=1}^{T} v_t v_t'$

- $\frac{1}{n}trS \xrightarrow{p} \sigma_v^2$
- $\|A\|_F^2 = tr\left(A'A\right)$, Frobenius norm
- Distance $(\frac{1}{n}trS)^{-1}S - I_n$: sample version of $(\sigma_v^2)^{-1}\Sigma_n - I_n$
- Locally the most powerful test for fixed n

Limiting Distribution of the Sphericity Tests Under the Null

- Fixed n : John (1972) as $T \to \infty$

$$\frac{nT}{2} U \xrightarrow{d} \chi^2_{n(n+1)/2-1} \tag{4}$$

- Large n : (Using Random Matrix Theory), Ledoit and Wolf (2002) as $(n, T) \to \infty$

$$TU - n \xrightarrow{d} N(1, 4) \tag{5}$$

  - Sphericity statistic J_0

$$J_0 = \frac{TU - n}{2} - \frac{1}{2} \xrightarrow{d} N(0, 1) \tag{6}$$

  - Ledoit, O., and Wolf, M. (2002), "Some Hypothesis Test for the Covariance Matrix When the Dimension is Large Compared to the Sample Size," *Annals of Statistics*, 30, 1081-1102.

Testing Cross-sectional Dependence in FE Models

- The error v_t in FE models: unobservable
- Residual-based statistic $\hat{J}_0$ using residuals $\hat{v}_t$

$$\hat{J}_0 = \frac{T\hat{U} - n}{2} - \frac{1}{2} \tag{7}$$

where $\hat{U} = (\frac{1}{n} tr\hat{S})^{-2} \frac{1}{n} tr\hat{S}^2 - 1$ and $\hat{S} = \frac{1}{T}\sum_{t=1}^{T} \hat{v}_t \hat{v}_t'$

- Can $\hat{J}_0$ be applied for testing cross-sectional dependence in a FE model? NO!
- The John statistic J in FE models

$$J = J_0 + (\hat{J}_0 - J_0) - \frac{n}{2(T-1)} = \hat{J}_0 - \frac{n}{2(T-1)}$$

  - $(\hat{J}_0 - J_0) - \frac{n}{2(T-1)} \xrightarrow{p} 0$
  - $J \xrightarrow{d} N(0,1)$

Asymptotics of the Bias Term under the Null

- Bias Term $\hat{J}_0 - J_0$
  - (No FE) $\mu_i = 0$: OLS residuals
  - (FE) $\mu_i \neq 0$: within residuals
- **Proposition**: Under Assumptions 1, 2 and 3,

(a) if $\mu_i = 0$,

$$\hat{J}_0 - J_0 = o_p(1); \tag{8}$$

(b) if $\mu_i \neq 0$,

$$\hat{J}_0 - J_0 - \frac{n}{2(T-1)} = o_p(1). \tag{9}$$

▸ Asymptotics

Histograms

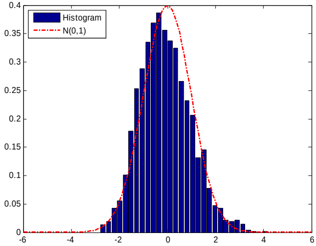

Figure 1 (a): The Histogram of the Residual-based Statistic $\hat{J}_0$ under the Null in the Case of no Fixed Effects

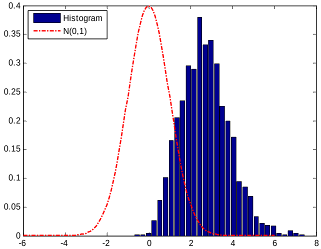

Figure 1 (b): The Histogram of the Residual-based Statistic $\hat{J}_0$ under the Null in the Case of Fixed Effects

Finite Bias Adjustment and Micro Panels

- The bias $\frac{n}{2(T-1)} \to 0$ for fixed n
- Size distortion due to large n
- In micro panels with large n and small T,

$$\hat{J}_0 - J_0 = \frac{\frac{n}{T}\sigma_v^8 - \frac{n}{T^2}\sigma_v^8 + O_p(\frac{1}{\sqrt{n}}) + O_p(\frac{\sqrt{n}}{T}) + O_p(\frac{1}{\sqrt{T}})}{\frac{2(T-1)^2}{T^2}\sigma_v^8 + O_p(\frac{1}{\sqrt{nT}})} \simeq \frac{n}{2(T-1)} \quad (10)$$

Bias: Incidental Parameters Problem

- The error v_{it} not identified directly in FE models: within residuals
- Inaccuracy in the within residuals accumulates in $\hat{J}_0 - J_0$
    - (No FE) $\mu_i = 0$: using OLS residuals $\widehat{v}_{it}$

$$\widehat{v}_{it} - v_{it} = -x_{it}(\widehat{\beta} - \beta) = O_p\left((nT)^{-1/2}\right) \tag{11}$$

"sample size": $n \cdot T$

    - (FE) $\mu_i \neq 0$: using within residuals $\widehat{v}_{it}$

$$\widehat{v}_{it} - v_{it} = \bar{v}_{i.} - \tilde{x}_{it}(\tilde{\beta} - \beta) = O_p(T^{-1/2}) + O_p\left((nT)^{-1/2}\right) = O_p(T^{-1/2}) \tag{12}$$

"sample size": T

Bias

Monte Carlo Simulations

- John test, compared with Pesaran's CD test and LM test
- Data Generating Process (DGP):

$$
\begin{aligned}
y_{it} &= \alpha + \beta x_{it} + \mu_i + v_{it}, \;\; i = 1, \cdots, n; \;\; t = 1, \cdots, T, && (13) \\
x_{it} &= 0.7 x_{i,t-1} + \mu_i + \eta_{it}, && (14)
\end{aligned}
$$

- $\mu_i \sim iidN(\phi_\mu, \sigma_\mu^2)$. $\eta_{it} \sim iidN(\phi_\eta, \sigma_\eta^2)$.
  - under H_0: $v_{it} \sim iidN(0, \sigma_v^2)$
  - $n = 5, 10, 20, 50, 100, 200, 500$ and $T = 5, 10, 20, 30, 50$
  - the replication number $R = 2000$.
- The values of parameters have no effect on the size
  - $\alpha = 1, \beta = 2, \phi_\mu = 0, \sigma_\mu^2 = 0.25, \sigma_v^2 = 0.5, \phi_\eta = 2, \sigma_\eta^2 = 1$

Empirical Size

Table 1: Size of Tests

Size	$T \setminus n$	5	10	20	30	50	100	200	500
John	5			9.7	10.5	12.4	12.1	11.9	12.7
	10	5.9	7.5	8.2	6.7	8.4	8.5	8.7	8.7
	20	5.5	7.3	5.5	6.5	6.6	7.1	6.7	6.6
	30	5.7	5.5	6.3	7.0	5.8	6.5	6.3	6.3
	50	5.0	7.0	7.0	6.9	5.3	5.8	5.7	4.1
Pesaran's CD	5			5.6	5.7	6.1	6.3	7.4	5.9
	10	5.2	5.4	5.3	5.2	5.0	5.3	5.4	4.9
	20	4.8	5.7	4.9	5.9	5.8	4.6	6.2	4.9
	30	6.2	4.9	4.7	3.8	5.5	5.1	5.6	5.1
	50	4.7	6.0	5.3	4.6	5.1	5.8	4.9	4.1
LM	5			68.8	100.0	100.0	100.0	100.0	100.0
	10	4.8	7.9	16.8	33.7	78.9	100.0	100.0	100.0
	20	4.1	4.9	7.2	10.7	23.9	74.2	100.0	100.0
	30	4.8	5.0	5.2	8.2	15.0	38.4	94.0	100.0
	50	4.2	5.6	6.6	6.5	8.1	17.5	50.9	100.0

Notes: The data generating process is specified in Section 6.

Power: 3 Models under the Alternative

- Factor model:

$$v_{it} = \gamma_i f_t + \varepsilon_{it}, \tag{15}$$

where factor $f_t \sim iidN(0,1)$, loading $\gamma_i \sim iidU(-0.5, 0.5)$, $\varepsilon_{it} \sim iidN(0, \sigma_v^2)$.

- Spatial autoregressive specification: SAR(1)

$$v_t = 0.4Wv_t + \varepsilon_t$$

  - W: spatial weight matrix 1 ahead and 1 behind

- Spatial moving average: SMA(1)

$$v_t = 0.4W\varepsilon_t + \varepsilon_t$$

Size-adjusted Power: Factor model

Table 2 (a) : Size Adjusted Power of Tests: the alternative is a factor model

Size Adjusted Power	$T \setminus n$	5	10	20	30	50	100	200	500
John	5			12.2	18.9	27.4	46.8	63.8	81.5
	10	7.9	15.1	27.7	45.9	63.4	84.2	94.6	98.9
	20	14.4	29.6	61.9	79.3	91.5	99.0	100.0	100.0
	30	17.8	48.6	80.6	91.2	99.2	100.0	100.0	100.0
	50	37.3	72.7	95.1	99.4	100.0	100.0	100.0	100.0
Pesaran's CD	5			4.4	3.8	3.5	4.7	5.4	5.3
	10	7.8	6.9	5.9	5.8	6.6	5.2	5.6	4.6
	20	7.5	7.6	7.5	6.7	7.2	7.2	7.2	7.0
	30	8.2	8.5	7.8	10.4	7.5	7.3	8.6	8.3
	50	12.8	11.9	10.4	12.0	10.6	11.1	11.9	12.5
LM	5			9.6	14.5	18.3	38.6	55.6	68.5
	10	9.1	12.4	24.5	36.9	56.0	79.2	91.2	98.4
	20	13.6	27.9	52.9	74.4	88.9	98.7	100.0	100.0
	30	16.3	41.2	76.1	89.0	98.2	100.0	100.0	100.0
	50	31.7	67.0	94.1	99.1	100.0	100.0	100.0	100.0

Notes: The data generating process is specified in Section 6.

Size-adjusted Power: SAR(1)

Table 2 (b) : Size Adjusted Power of Tests: The alternative is a SAR (1) model

Size Adjusted Power	$T \setminus n$	5	10	20	30	50	100	200	500
John	5			11.4	11.7	12.55	13.25	13.85	13.55
	10	22.7	24.4	26.3	31.2	32.9	32.5	33.8	31.6
	20	59.5	65.0	78.5	81.6	81.8	82.4	84.1	84.1
	30	84.5	94.7	96.6	97.4	99.2	99.4	99.6	99.2
	50	99.3	99.9	100.0	100.0	100.0	100.0	100.0	100.0
Pesaran's CD	5			24.4	24.8	25.0	22.9	25.3	24.7
	10	56.6	48.0	41.4	46.6	44.3	41.9	40.5	39.3
	20	82.9	75.5	70.8	68.8	74.3	69.8	69.1	70.5
	30	95.0	87.5	86.1	88.3	87.8	84.8	82.3	85.3
	50	99.7	98.2	97.6	97.4	96.2	97.5	97.3	95.9
LM	5			9.0	10.7	7.8	8.8	10.6	9.0
	10	30.3	19.6	25.6	55.2	26.4	27.2	26.0	27.0
	20	66.9	73.2	75.2	78.9	75.9	79.9	80.6	77.5
	30	90.4	95.5	97.5	97.2	98.8	98.8	99.0	98.8
	50	99.7	100.0	100.0	100.0	100.0	100.0	100.0	100.0

Notes: The data generating process is specified in Section 6.

Size-adjusted Power: SMA(1)

Table 2 (c) : Size Adjusted Power of Tests: The alternative is a SMA (1) model

Size Adjusted Power	$T \setminus n$	5	10	20	30	50	100	200	500
John	5			8.5	10.6	9.2	10.3	10.1	10.3
	10	15.4	19.8	18.3	21.4	22.3	25.3	24.1	21.5
	20	47.1	50.5	63.3	66.5	63.9	65.0	66.5	68.1
	30	72.2	83.6	90.2	91.1	94.6	95.4	95.3	95.3
	50	98.9	99.8	99.9	100.0	100.0	100.0	100.0	100.0
Pesaran's CD	5			16.4	18.4	19.4	16.5	17.7	17.9
	10	38.6	37.1	28.4	31.5	31.4	28.7	26.6	25.0
	20	64.0	56.0	51.8	50.3	52.2	51.4	47.7	51.7
	30	82.2	70.6	66.7	73.0	70.5	66.9	65.9	67.7
	50	95.5	90.6	89.8	86.6	85.5	86.5	86.4	86.3
LM	5			6.5	9.7	7.0	7.5	9.3	8.2
	10	18.3	15.9	17.8	16.1	17.8	17.5	18.7	19.9
	20	53.9	58.4	59.5	63.7	57.2	63.4	62.9	60.3
	30	82.8	85.7	92.2	90.9	91.9	94.4	93.6	94.3
	50	99.6	99.9	100.0	100.0	100.0	100.0	100.0	100.0

Notes: The data generating process is specified in Section 6.

Conclusion

- A new test for cross-sectional dependence in FE models: the John test
- Desirable finite sample properties
- Successfully control the size distortion for large n
- Applicable in micro panels

Extensions:

- Allow for heterokedasticity across the cross-sectional units (in progress)
- Power properties

Thank You!

Asymptotics of the Bias Term under the Null:

- The bias term:

$$\hat{J}_0 - J_0 = \frac{T(\hat{U} - U)}{2} = \frac{T\left[\left(\frac{1}{n}trS\right)^2 \frac{1}{n}tr\hat{S}^2 - \left(\frac{1}{n}tr\hat{S}\right)^2 \frac{1}{n}trS^2\right]}{2\left(\frac{1}{n}tr\hat{S}\right)^2 \left(\frac{1}{n}trS\right)^2}$$

- 4 terms: $\frac{1}{n}trS$, $\frac{1}{n}trS^2$, $\frac{1}{n}tr\hat{S} - \frac{1}{n}trS$ and $\frac{1}{n}tr\hat{S}^2 - \frac{1}{n}trS^2$

Asymptotics: First Moment Difference

- Ledoit and Wolf (2002): As $(n, T) \to \infty$ with $\frac{n}{T} \to c \in (0, \infty)$

$$\frac{1}{n} trS \xrightarrow{p} \sigma_v^2 \text{ and } \frac{1}{n} trS^2 \xrightarrow{p} (1+c)\left(\sigma_v^2\right)^2.$$

- **Proposition 1**: Under Assumptions 1 and 2,

(a) if $\mu_i = 0$, using OLS residuals $\frac{1}{n} tr\widehat{S} - \frac{1}{n} trS = O_p(\frac{1}{nT})$;
(b) if $\mu_i \neq 0$, using within residuals $\frac{1}{n} tr\widehat{S} - \frac{1}{n} trS = O_p(\frac{1}{T})$

- No relationship between n and T (Assumption 3) needed
  - For fixed n, (a) and (b) have the same result
  - For large n, (a) and (b) are different

Asymptotics: Second Moment Difference

- **Proposition 2**: Under Assumptions 1 and 2

(a) if $\mu_i = 0$, using OLS residuals

$$\frac{1}{n} tr\widehat{S}^2 - \frac{1}{n} trS^2 = O_p(\frac{1}{T\sqrt{nT}}) + O_p(\frac{1}{T^2}) + O_p(\frac{1}{nT}), \qquad (16)$$

(b) if $\mu_i \neq 0$, using within residuals

$$\frac{1}{n} tr\widehat{S}^2 - \frac{1}{n} trS^2 = O_p(\frac{1}{T}) + O_p(\frac{n}{T^2}). \qquad (17)$$

Return

Two cases: no FE vs FE

- $\widehat{S} - S = \frac{1}{T}\sum_{t=1}^{T} \widehat{v}_t \widehat{v}_t' - \frac{1}{T}\sum_{t=1}^{T} v_t v_t'$
  - no FE: $\widehat{v}_t$ OLS residuals
  - FE: $\widehat{v}_t$ within residuals

(a) No FE:

$$\widehat{S} - S = -\frac{1}{T}\sum_{t=1}^{T} x_t(\widehat{\beta} - \beta)v_t' - \frac{1}{T}\sum_{t=1}^{T} v_t(\widehat{\beta} - \beta)'x_t' + \frac{1}{T}\sum_{t=1}^{T} x_t(\widehat{\beta} - \beta)(\widehat{\beta} - \beta)'x_t' \qquad (18)$$

(b) FE:

$$\widehat{S} - S = -\frac{1}{T}\sum_{t=1}^{T} \tilde{x}_t(\widehat{\beta} - \beta)\tilde{v}_t' - \frac{1}{T}\sum_{t=1}^{T} \tilde{v}_t(\widehat{\beta} - \beta)'\tilde{x}_t' + \frac{1}{T}\sum_{t=1}^{T} \tilde{x}_t(\widehat{\beta} - \beta)(\widehat{\beta} - \beta)'\tilde{x}_t'. - \bar{v}.\bar{v}.' \qquad (19)$$

First and Second Moment Difference: no FE and FE

- An extra term $-\bar{v}_{.}\bar{v}_{.}'$ in FE models

$$\frac{1}{n}tr(-\bar{v}_{.}\bar{v}_{.}') = -\frac{1}{n}\sum_{i=1}^{n}\bar{v}_{i.}^{2} = -\frac{1}{n}\sum_{i=1}^{n}\left(\frac{1}{T}\sum_{t=1}^{T}v_{it}\right)^{2} = O_p(\frac{1}{T}).$$

- First moment difference $\frac{1}{n}tr\widehat{S} - \frac{1}{n}trS$:
  - $\frac{1}{n}trS$: first moment
  - No FE: $\frac{1}{n}tr\widehat{S} - \frac{1}{n}trS = O_p(\frac{1}{nT})$
  - FE: $\frac{1}{n}tr\widehat{S} - \frac{1}{n}trS = O_p(\frac{1}{T}) + O_p(\frac{1}{nT}) = O_p(\frac{1}{T})$
  - Difference: the extra term due to the within transformation
- Second moment difference $\frac{1}{n}tr\hat{S}^2 - \frac{1}{n}trS^2$: similar

Return

Robustness check: DGP without FE

Table 3 : Size of Tests: Robustness to the case of no fixed effects

Size	T \ n	5	10	20	30	50	100	200	500
John	5			9.7	9.8	13.3	12.8	12.4	10.6
	10	6.2	6.7	7.9	7.3	8.4	8.3	8.8	7.5
	20	5.4	6.4	7.0	6.9	6.6	6.8	7.7	7.6
	30	5.6	4.7	6.1	6.2	6.0	6.4	5.7	5.6
	50	4.6	5.8	7.3	6.0	5.5	6.6	5.2	6.2
Pesaran's CD	5			6.2	6.4	5.9	6.2	7.5	6.8
	10	4.3	4.4	4.8	4.7	5.5	5.6	5.6	4.6
	20	4.6	4.3	5.4	5.0	5.4	4.6	5.5	4.7
	30	4.2	4.5	4.9	5.1	6.3	5.5	4.6	4.9
	50	5.5	5.0	4.6	5.0	5.3	5.2	5.7	4.1
LM	5			68.7	100.0	100.0	100.0	100.0	100.0
	10	6.0	7.4	16.0	32.8	80.1	100.0	100.0	100.0
	20	4.4	6.1	7.5	11.2	25.1	73.8	100.0	100.0
	30	5.1	5.0	5.3	7.2	12.2	38.8	92.6	100.0
	50	4.6	4.3	6.2	5.8	7.1	18.1	51.9	100.0

Notes: The data generating process is specified in Section 6.